Forbidden Formulas: Elitism in Math

Breaking Math

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1 Introduction

From Pythagoras to Einstein, from the banks of the Nile to the streamlined curves of the Large Hadron Collider, math has shown itself again and again to be fundamental to the way that humans interact with the world. Then why is math such a pain for so many people? Our answer is simple: math is, and always has been, in one way or another, guarded as an elite skill. We visit the worlds that were shaped by math, the secrets people died for, the false gods created through this noble science, and the gradual chipping away of this knowledge by a people who have always yearned for this magical skill. So what is it? And how can we make it better?

2 Genesis of Elitism

As we use it, "elitism" refers to a resource, or in this case a skill, that is primarily available to a select group of people. For example. one modern example of the way in which elitism pervades mathematics is seen in Wikipedia articles about mathematics; arguably, many articles about mathematics are only accessible to those who already have had these skills. It can be argued, of course, that this is a failure of autopedagogy, but it is the editorial opinion of Breaking Math that this is not necessarily the case.

So why is mathematics a resource that is jealously guarded? Essentially, because it is useful. Industries have been made or broken by mathematics. One very early example involves farming on the banks of the Nile river. Those who were able to watch the stars and predict floods were able to plan for farming more efficiently, and consequently, they were able to reap more grain. It was, in a phrase, a boon to agricultural business.

3 Ancient Greece

Right across the mediterranean from the kingdom of Egypt lay a very different sort of place, where ideas, rather than dynasties, distinguished subculture from subculture. This was, of course, the land of ancient Greece. But not all was rosy in such a place. People suffered, fought, and even spilled blood to defend and control ideas which would seem innocuous to the modern mind.

Ancient Greek mathematics was much more complex than Egyptian mathematics. One reason for this position is the Greek emphasis on rigor versus empiricism, or from application to theory. Egyptians were more concerned with borders and floods, but the Greeks also studied mathematics for the sake of mathematics. Mathematics has continued largely in this tradition, with benefits being reaped, in many cases, much later than the purely mathematical discovery (number theory was studied in ancient Greece, and feverously in the 1800s, but did not have many applications until cryptography became necessary for credit card transactions and the like). The most famous example of ancient Greek rigor was Euclid (roughly 350BCE to 250BC), who published his geometry tome *Elements*; so valuable were the proofs contained therein that they still challenge and inspire students to this day. A quick example of this sort of mathematical reasoning will be given.

A full diagram for the following problem is given in *fig. 1*. The problem is found in Euclid's *Elements* in Book I, Proposition V, and is known as the *Pons Asinorium* or the *Bridge of Asses*. It's known as such for its elusive logic. The goal of this problem is to prove that the angles of an isosceles triangle (one with two sides equal to one another) that are next to the sides which equal one another are equal to each other. This seems like something that one would take for granted, but Euclid thought otherwise.

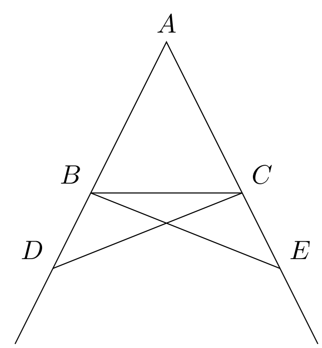

Figure 1: The Bridge of Asses

The setup is as follows: draw a triangle with A, B, and C such that $\overline{AB}$ (that is to say, the line between A and B) is equal to AC. The claim can now be phrased that $\angle ABC$ (the angle centered at B) and $\angle ACB$ are equal to one another.

Extend the line AB, and take an arbitrary point D on it. Now extend the line AC and put a point E on it, such that $\overline{AE}$ equals $\overline{AD}$. We now have two new triangles $\triangle ACD$ and $\triangle ABE$, which are just mirror images of one another, since $\overline{AD}$ equals $\overline{AE}$ and $\overline{AB}$ equals $\overline{AC}$, we know that $\overline{BD}$ equals $\overline{CE}$ (since "equals subtracted from equals are equal", and the other sides $\overline{AD}$ and $\overline{AE}$ are also equal, and they share the angle $\angle DAC = \angle BAE = \angle BAC$, and it was proven earlier in the book that if two triangles share a side, angle, and a side, then they are equal). Since the triangles are equal, we know that $\overline{BE}$ and $\overline{CD}$ are equal. Since $\overline{BC}$, $\overline{CE}$, and $\overline{BE}$ equal $\overline{BC}$, $\overline{CD}$, and $\overline{BD}$ respectively, we know that $\triangle BCD$ equals $\triangle CBE$. Now, since $\triangle ACD = \triangle ABE$, we know that $\angle ABE = \angle ACD$. Similarly, since $\triangle BCD = \triangle CBE$, we know that $\angle CBD = \angle BCE$. Since "equals subtracted from equals are equals", and $\angle ABE - \angle CBD = \angle ABC$ and $\angle ACD - \angle BCD = \angle ACB$, we know that $\angle ABC = \angle ACB$, and thus it has been proven.

Euclid's *Elements* contains 393 such proofs in total, and each is proven rigorously, and they have all stood the test of time (nonwithstanding noneuclidean geometry). An earlier thinker, however, was Pythagoras (570BCE to 495BCE). Incorrectly famous for the theorem which bears his name (the Pythagorean theeorem, which was almost certainly taught to him), and famous in the medieval mind for the invention of the musical scale, he lived an interesting life as the leader of a vegan cult which regarded beans as evil (for their flatulent properties).

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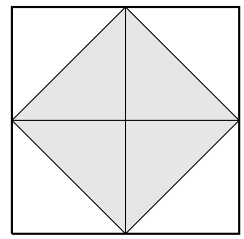

Figure 2: Pythagoras' lesson for Meno's slave

An example of how even this seemingly noble Greek tradition was imbued with the corruption of elitism can be seen in the story of Meno's slave. The story is thus (and illustrated in *fig. 2*. There was a man called Meno who had a slave, and argued with Socrates (470/469BCE to 399BCE) about whether or not slaves could learn mathematics; Meno purported that they intrinsically could not, and Socrates held that this was not the case. To defend his position, Socrates decided to teach Meno's slave a lesson in mathematics. In the ground, he drew a square, and asked the slave to draw a square that was twice its size. The slave drew three squares on three adjacent sides of the original square, resulting in a 2×2 square. Socrates then gently corrected the slave, and proceeded to draw diagonals between the four squares that were already there. He showed the slave that since each diagonal cut each square in half, the resulting square had to be four times half of the size of the original square; or, twice the size. When asked to recite the lesson, the slave was able to do so.

Of course, no original writings by Socrates were either made or have survived, so we only know of this story from Plato (428/427BCE or 424/423BCE to 348/347BCE), who was known to take liberties with the truth. Whether or not this was Socrates fanfiction, the attitude conveyed with respect to the mental capacities of the non-elite (in this case, slaves) survive through this Socratic dialogue, and continued well past Plato's time (it was even argued that slaves who did not like to work suffered from a pseudoscientic disorder known as "drapetomania" in the 19^{th} century).

In his own way, Pythagoras perpetuated an attitude of elitism. Pythagoras made knowledge of certian mathematical concepts within his cult privileged information, to the point that divulging them to outsiders was cause for capital punishment. One such concept was the existence of irrational numbers (numbers that cannot be represented as the ratio of two whole numbers), which challenged the perfection of the natural (whole) numbers. Hippasus is known (as well as knowledge from this era can be accurately known) to have met such a fate. The stories vary from drowning to beheading, but the message persists.

4 The Dark Ages and the Renaissance

Along with centuries of time betwen ancient Greece and medieval Euroope came a deep deficit of knowledge. Engineering, cultural, and mathematical knowledge was lost to the west, and rare books were bleached to rescribe common ones, and all knowledge of curatorship was preserved by a loose network of monestaries.

It was in the latter half of that history that the Peripatetic, or Aristotelean, view dominated (and, in fact, was backed up by the medieval church and enforced by states). It was in this environment that thinkers such as Gerbert of Aurillac (955CE to 1003CE), who was accused of convening with the devil; and Galileo Galilei (1564CE to 1642CE), who was accused of heresy, struggled against elitism: although in Europe there was a dearth of advancement, the control of knowledge was absolutely not unknown to thinkers of the middle ages.

We will begin with a quote from Galileo's *A Dialogue Concerning the two Chief World Systems*, a book for which he was persecuted:

> One day I was at the home of a very famous doctor in Venice where many persons came on account of their studies, and others occasionally came out of curiosity to see some anatomical dissection performed by a man who was truly no less learned than he was a careful and expert anatomist. It happened on this day that he was investigating the source and origin of the nerves, about which there exist a notorious controversy between the Galenist and Peripatetic [Aristotelean] doctors. The anatomist showed that the great trunk of nerves leaving the brain and passing through the nape extended on down the spine and then branched out through the whole body, and only a single strand as fine as a thread arrived at the heart. Turning to a gentlement whom he knew to be an Aristotelean philosopher, and on whose account he had been exhibiting and demonstrating everything with unusual care, he asked this man whether he was at last satisfied and convinced that the nerves originated in the brain, and not in the heart. The philosopher, after considering for a while, answered, "You have made me see this matter so plainly and so palpably that if Aristotle's texts were not contrary to it, stating clearly that the nerves originate in the heart, I should be forced to admit it to be true.

The control of knowledge is apparent: Aristotle was so revered as perfect truth that knowledge was made almost impossible to attain. Of course, our current world is proof that this was not a perfect seal, but the duration of the dark and middle agest is testament to the corruption inherent to elitism. We may laugh at these examples, but mathematics is still taught as a fact, not as an invention. We may have, and have had in the past, biases with regard to the way we view mathematics because we see something as obvious. Euclid, as brilliant as he was, did not consider the geometry created by not assuming that two straight lines may intersect (as seen on the surface of a sphere), and mathematicians in the 19$^{\text{th}}$ century went through a small crisis when their beloved theory of sets (a set is just a group of things, including sets) was shown to be logically fallible (the question they asked, to paraphrase, was "does a group of things which don't contain themselves contain itself?").

Going back a few centuries, we come to Gerbert of Aurillac, who was born in the dark ages, and studied in Moorish Spain, where he learned arabic numerals, which were far superior to the Roman numerals with which his peers were more well-acquainted. The speed with which he was able to compute (both mentally and otherwise) with these symbols earned him a reputation for convening with the devil. He was even rumored to have a robotic head which gave him answers. This is an example of how elitism can take the form of existing biases; in this case, xenophobia. Only because of his wealth and eventual ascention to the papacy as Pope Sylvester II was he able to escape persecution. Others were not so lucky; merchants in Venice were, even centuries later, banned from using them in their books. In this time, Roman numeerals were used to compute. The abaciss, as they were known in the advent of the algorists (those who used Arabic numerals), used boards with pebbles on them to laboriously compute sums. Even this form of elitism persists to this day in the United States, which has not followed the rest of the world in metrification (which has led to at least one spaceship disaster).

5 Modern Elitism

Jakow Trachenberg (1888 to 1953) was a Russian man who developed a system of speed math, known as Trachtenberg Speed Math, to keep his mind occupied while being held in a Nazi concentration camp.

One method of rapid addition in Trachtenberg Speed Math is known as “casting out elevens”. The method is simple, and is illustrated in *fig. 3*. One writes the numbers in columns, and adds like normal, but every time a number greater than or equal to 11 is reached, a tick (’) is put next to that number. The remainder is written down, and the number of ticks is written below it. In the example below, it goes 3, $3+4=7$, $7+7=14$ which turns into a 3 since $14-11=3$, 10, 11 (tick, 0), 4, 10, 19 (tick, 8), 10, 17 (tick, 6). Since we have a total of 4 ticks, the 6 is written down, and the 4 right below it. All of the columns are completed in this fashion. Next, the entries are summed in an L shape. The rightmost two are summed directly (one can imagine an invisible 0 next to the 4), and then the rest are summed in this manner (which has been highlighted for your convenience). Thus, the answer to the sum in the figure is 50, 842, 929, 060.

	3	1	4	1	5	9	2	6	5	3
	5	8	9’	7	9’	3’	2	3	8’	4
	6’	2’	6	4’	3	3	8’	3’	2	7’
	9’	5	0	2	8’	8’	4	1	9’	7
	1	6	9’	3	9’	9	3	7	5	1’
	0	5’	8’	2	0	9’	7’	4’	9’	4
	4	5	9’	2	3	0	7’	8	1	6
	4	0	6	2’	8’	6’	2	0	8’	9’
	9’	8’	6’	2	8	0	3	4’	8’	2
+	5’	3	4	2	1	1	7’	0	6	7’
	2	9	6	5	·0	4	1	1	6	6
↪	4	3	5	2	4	4	4	3	5	4
5	·0	·8	·4	·2	·9	·2	9	·0	·6	·0

Figure 3: Trachtenberg Speed Addition

Using this method, and others similar to it, Trachtenberg was able to advance remedial math students past their peers. Although this method is demonstrably (with limited evidence, admittedly) better than other methods, it has not been adopted by the educational system at large. This is an obvious example of modern elitism through the mechanism of stagnation.e